Topic → Problem based on linear Equation with constant Co-efficients

Class - B.Sc (Part II) Date -

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Problem →

Solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

Solution → Given Equation is

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

it may be written as

$$(D^2 + 2D + 1)y = 0$$

Auxiliary Equation

$$D^2 + 2D + 1 = 0$$

$$\Rightarrow (D+1)^2 = 0$$

$$D = -1, -1$$

Hence the General Solution

$$y = (C_1 + C_2 x)e^{-x}$$

Ex: → Solve

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$$

Solution Given Equation

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$$

This may be written as

$$(D^3 - 4D^2 + 5D - 2)y = 0$$

The Auxiliary Equation

$$D^3 - 4D^2 + 5D - 2 = 0$$

$\Rightarrow$ $D^3 - D^2 - 3D^2 + 3D + 2D - 2 = 0$

$\Rightarrow$ $D^2(D-1) - 3D(D-1) + 2(D-1) = 0$

$\Rightarrow$ $(D-1)(D^2 - 3D + 2) = 0$

$\Rightarrow$ $(D-1)(D-1)(D-2) = 0$

$D^2 - 3D + 2 = 0$

$D = \frac{3 \pm \sqrt{9-8}}{2}$

$= \frac{3 \pm 1}{2} = 2, 1$

$\therefore D = 1, 1, 2$

Hence the General Solution

$$(C_1 + C_2x)e^x + C_3e^{2x}$$

Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0$

Solution :→ The Given Equation

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0$$

it may be written as

$$(D^3 - D^2 + 4D - 4)y = 0$$

$\therefore$ Auxiliary Equation is

$$D^3 - D^2 + 4D - 4 = 0$$

$\Rightarrow$ $D^2(D-1) + 4(D-1) = 0$

$\Rightarrow$ $(D-1)(D^2+4) = 0$

$\Rightarrow$ either $D-1=0$ or $D^2+4=0$

$\Rightarrow D=1$ $\quad\Rightarrow D^2=4i^2$

$\Rightarrow D=\pm 2i$

$\therefore D=1,\ D=2i,\ D=-2i$

Hence the General ~~Equation~~ Solution

$C_1e^{x}+C_2\cos 2x+C_3\sin 2x$

$\left[\begin{array}{l} e^{i\theta}=\cos\theta+i\sin\theta \\ e^{-i\theta}=\cos\theta-i\sin\theta \end{array}\right]$

$\cancel{C_2 e^{2i} = C_2 \cos 2x + i\sin 2x}$

$\cancel{C_3 e^{-2i} \quad C_3 = \cos 2x - i\sin 2x}$

Solve $\frac{d^3y}{dx^3}-8y=0$

Solution :-> the Given Equation is

$$\frac{d^3y}{dx^3}-8y=0$$

it may be written as

$$(D^3-8)y=0$$

$\therefore$ Auxiliary Equation

$$D^3-8=0$$

$$(D-2)(D^2+2D+4)=0$$

either $D-2=0$ or $D^2+2D+4=0$

$\Rightarrow D=2$ $\quad$ or $D=\frac{-2\pm\sqrt{4-16}}{2}$

$\therefore$ General solution $\quad =\frac{-2\pm\sqrt{-12}}{2}$

$\cancel{C_1e^{2x}+C_2e^{x-2}(C_2\cos\sqrt{3}x+C_3\sin\sqrt{3}x)^2}$

$=\frac{-2\pm\sqrt{12i^2}}{2} = -1\pm\sqrt{3}i$

$D=2,\ -1+\sqrt{3}i,\ -1-\sqrt{3}i$

General Solution

$C_1e^{2x}+e^{-x}(C_2\cos\sqrt{3}x+C_3\sin\sqrt{3}x)$